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RANKING FUNCTION METHODS FOR SOLVING FUZZY LARGE-SCALE MULTI-OBJECTIVE INTEGER LINEAR FRACTIONAL PROGRAMMING PROBLEMS

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ABSTRACT

This paper presents a method for solving fuzzy large-scale multi-objective integer linear fractional programming problems. First, fuzzy large-scale multi-objective integer linear fractional programming problems are transformed into a crisp model, using various kinds of linear ranking functions. Then, the fractional functions are transformed into linear functions, using linearization technique. Second, the linearization model is solved to obtain an efficient solution for proposed problems. A numerical example is given to illustrate efficiency of the proposed method.

Keywords: Multiobjective linear programming; Integer programming; Decomposition algorithm; Fractional programming; Fuzzy set theory; Linear ranking function.

1. INTRODUCTION

The concept of fuzzy decision making was brought into programming research by Zimmerman [22] under the term fuzzy linear programming (FLP), and it has since been used in a variety of problem types [1, 10, 14-17]. Toksari [20] presented a method based on a a Taylor series for solving fuzzy multiple objective linear fractional programming problems. Dheyab [4] used the ranking functions to covert fractional linear programming problems in a fuzzy environment into a crisp model. Nehi and HajMohamadi [13] presented a method based on ranking function to solve fuzzy multi-objective linear programming problem. An improved method based on the branch and cut concept for solving multiobjective integer linear fractional programming problem was introduced by Mehdi et al. [12]. Bharati et al. [3] introduced a class of distance functions between two trapezoidal fuzzy numbers and used it to solve a fully fuzzy multiple objective linear programming (FFMOLP) problem. A fuzzy compromise solution for FFMOLP problems was developed by Hamadameen and Hassan [8]. Arana-Jimenez [2] presented a new method that was based on multiple objective linear programming (MOLP) to find a non-dominated solution to FFLP problems.

This paper is organized into five sections. In the next section, definitions of various kinds of linear ranking functions are reviewed. In Section 3, large-scale multi-objective integer linear fractional programming problems with fuzzy numbers in the constraints and with block angular structure (FLSMLFP) are formulated, and a detailed methodology is given to solve this type of problem. A illustrative numerical example for the proposed method is presented in Section 4. Finally, this paper is concluded in Section 5.

2. LINEAR RANKING FUNCTIONS

An efficient approach for ordering fuzzy numbers is to define a ranking function $D: F(\mathfrak{R}) \to \mathfrak{R}$ which is a mapping of the set of fuzzy numbers on a real line, where a natural order exists. Suppose that $\tilde{a}$ and $\tilde{b}$ be two trapezoidal fuzzy numbers in $F(\mathfrak{R})$, then the orders on $F(\mathfrak{R})$ are defined, as follows:

$$\tilde{a} \geq \tilde{b} \text{ if and only if } D(\tilde{a}) \geq D(\tilde{b}),$$
$$\tilde{a} \leq \tilde{b} \text{ if and only if } D(\tilde{a}) \leq D(\tilde{b}),$$
$$\tilde{a} = \tilde{b} \text{ if and only if } D(\tilde{a}) = D(\tilde{b}).$$

In this work, we use the linear ranking functions adopted by Maleki [11] and Yager [21] as follows:

2.1. Maleki Ranking Function [11]

The ranking function of a fuzzy number $\tilde{a} = (a^l, a^u, \alpha, \beta)$ is defined as follows:

$$D(\tilde{a}) = a^l + a^u + \frac{1}{2}(\beta - \alpha) \quad (1)$$

2.2. Yager Ranking Function [21]

The ranking function of a fuzzy number $\tilde{a} = (a^l, a^u, \alpha, \beta)$ is defined as follows:

$$D(\tilde{a}) = \frac{1}{2}\left[a^l + a^u - \frac{4}{5}\alpha + \frac{2}{3}\beta\right] \quad (2)$$

3. METHODOLOGY

3.1. Formulation of Fuzzy Large–Scale Multi-objective Integer Linear Fractional Programming Problem

Let us consider the following FLSMLFP problems:

$$\text{FLSMLFP}: max F(x) = \{f_1(x), f_2(x), \dots, f_k(x)\}, \quad (3.a)$$

subject to

$$\sum_{j=1}^{n} a_{0j} x_j \leq \tilde{b}_0, \quad (3.b)$$

$$\textstyle\sum_{j=1}^{n} a_{ij}x_j \leq \tilde{b}_i \ (i = 1,2, \dots, m) \qquad (3.c)$$
$$x_j \geq 0 \ and \ integer \ for \ all \ j = 1,2, \dots, n. \qquad (3.d)$$

where $x \in R^n$ is a vector of decision variables, $\tilde{b} = (\tilde{b}_0, \tilde{b}_1, \dots, \tilde{b}_m)$ is an $(m+1)$ fuzzy vector, a_{0j} and a_{ij} $(i = 1,2, \dots, m; j = 1,2, \dots, n)$ are constants.

The ith linear fractional objective function takes the form:

$$f_i(x) = \frac{\sum_{j=1}^{n} c_{ij}x_j + \lambda_i}{\sum_{j=1}^{n} d_{ij}x_j + \gamma_i} \ (i = 1,2, \dots, k) \qquad (4)$$

3.2. Crisp Model of FLSMLFP Problems

The basic idea in treating FLSMLFP problems is to define a crisp model equivalent to the proposed problem. The idea of employing deterministic version will be illustrated by using two ranking functions, Maleki [11] and Yager [21].

3.2.1. Using Maleki Ranking Function [11]

$$\textstyle\sum_{j=1}^{n} a_{0j}x_j \leq \left[b_0^l + b_0^u + \frac{1}{2}(\beta_0 - \alpha_0)\right], \qquad (5.a)$$
$$\textstyle\sum_{j=1}^{n} a_{ij}x_j \leq \left[b_i^l + b_i^u + \frac{1}{2}(\beta_i - \alpha_i)\right], \qquad (5.b)$$

where $\tilde{b}_0 = (b_0^l, b_0^u, \alpha_0, \beta_0)$ and $\tilde{b}_i = (b_i^l, b_i^u, \alpha_i, \beta_i)$ $(i = 1,2, \dots, m)$ are trapezoidal fuzzy numbers.

3.2.2. Using Yager Ranking Function [21]

$$\textstyle\sum_{j=1}^{n} a_{0j}x_j \leq \frac{1}{2}\left[b_0^l + b_0^u - \frac{4}{5}\ \alpha_0 + \frac{2}{3}\beta_0\right], \qquad (6.a)$$
$$\textstyle\sum_{j=1}^{n} a_{ij}x_j \leq \frac{1}{2}\left[b_i^l + b_i^u - \frac{4}{5}\ \alpha_i + \frac{2}{3}\beta_i\right], (i = 1,2, \dots, m) \qquad (6.b)$$

Now, the crisp large–scale multiobjective integer linear fractional programming (LSMLFP) problems equivalent to FLSMLFP problems can be written, as follows:

$$\text{LSMLFP}: max f_i(x) = \frac{\sum_{j=1}^{n} c_{ij}x_j + \lambda_i}{\sum_{j=1}^{n} d_{ij}x_j + \gamma_i} \ (i = 1,2, \dots, k), \qquad (7.a)$$

subject to

$$x \in S_M = \left\{ x \in R^n \middle| \begin{array}{c} \sum_{j=1}^{n} a_{0j}x_j \leq \left[b_0^l + b_0^u + \frac{1}{2}(\beta_0 - \alpha_0)\right], \\ \sum_{j=1}^{n} a_{ij}x_j \leq \left[b_i^l + b_i^u + \frac{1}{2}(\beta_i - \alpha_i)\right], \\ x_j \geq 0 \ and \ integer \ for \ all \ j. \end{array} \right\} \qquad (7.b)$$

or

$$x \in S_Y = \left\{ x \in R^n \middle| \begin{array}{l} \sum_{j=1}^{n} a_{0j} x_j \le \frac{1}{2}\left[b_0^l + b_0^u - \frac{4}{5}\,\alpha_0 + \frac{2}{3}\beta_0\right], \\ \sum_{j=1}^{n} a_{ij} x_j \le \frac{1}{2}\left[b_i^l + b_i^u - \frac{4}{5}\,\alpha_i + \frac{2}{3}\beta_i\right], \\ \quad x_j \ge 0 \; and \; integer \; for \; all \; j. \end{array} \right\} \tag{7.c}$$

Definition 3.1. Let $\tilde{S}$ be the set of all crisp feasible solution of FLSMLFP problems. Then $x^* \in \tilde{S}$ is said to be an efficient optimal solution for FLSMLFP problems if there does not exist another $x_j \in \tilde{S}$ $(j = 1,2, \dots, n)$ such that $F(x) > F(x^*)$ and $F(x) \ne F(x^*)$.

3.2. Linearization Model of LSMLFP Problems

Here, the fractional linear function from each objective is converted to a linear polynomial using the 1st order Taylor series [18]. The transformation of the objective functions $f_i(x), (i = 1,2, \dots, k)$ to polynomial functions is given in the following form:

$$f_i(x) \cong f_i(x_i^*) + \sum_{j=1}^{n} \left(x_j - x_{ij}^*\right) \frac{\partial f_i(x_i^*)}{\partial x_j} (i = 1,2, \dots, k) \tag{8}$$

where $x_i^* = (x_{i1}^*, x_{i2}^*, \dots, x_{in}^*)$ is the value that is used to maximize the ith objective function $f_i(x)$ $(i = 1,2, \dots, k)$ and *n* is a number of variables.

Then, using the nonnegative weighted sum approach [9], LSMLFP problems can be transformed into large-scale integer linear programming problems with single objective function (LSLP) as follows:

$$\text{LSLP}: max \sum_{i=1}^{k} w_i \left(f_i(x_i^*) + \sum_{j=1}^{n} \left(x_j - x_{ij}^*\right) \frac{\partial f_i(x_i^*)}{\partial x_j} \right), \tag{9}$$

subject to

$$x \in S_M \; or \; x \in S_Y$$

where $w_i \ge 0$ $(i = 1,2, \dots, k)$ and $\sum_{i=1}^{k} w_i = 1$.

3.4. Decomposition Algorithm for LSLP Problems

Using the decomposition algorithm [5, 19], LSLP problems are broken into n-sub problems that can be solved independently. Assuming that each of the convex set of (5.b), or (6.b), is bounded, then every point in this set can be expressed as a convex combination of the extreme points i.e.,

$$x_j = \sum_{k=1}^{k_j} \beta_j^k \, \hat{x}_j^k, (j = 1,2, \dots, n) \tag{10.a}$$

$$\beta_j^k \ge 0 \text{ and } \sum_{k=1}^{k_j} \beta_j^k = 1 \text{ for all } j. \tag{10.b}$$

where k_j is a number of extreme points of set *j* and $\hat{x}_j^k$ $\left(k = 1, \dots, k_j\right)$ are the extreme points of the *j*th set.

Ignoring integer conditions reduce LSLP problems to modified problem in terms of β_j^k, as follows:

$$\hat{P}(x): max \sum_{j=1}^{n} \hat{C}_j \left(\sum_{k=1}^{k_j} \beta_j^k \hat{x}_j^k\right), \tag{11.a}$$

subject to

$$x \in \hat{S}_M = \begin{cases} \sum_{k=1}^{k_j} a_{ij}\beta_j^k \hat{x}_j^k \leq \left[b_i^l + b_i^u + \frac{1}{2}(\beta_i - \alpha_i)\right] & (i = 1,2,\dots,m) \\ \sum_{k=1}^{k_j} \beta_j^k = 1, & (j = 1,2,\dots,n) \\ \quad \beta_j^k \geq 0 \; for\ all\ j\ and\ k \end{cases} \tag{11.b}$$

or

$$x \in \hat{S}_Y = \begin{cases} \sum_{k=1}^{k_j} a_{ij}\beta_j^k \hat{x}_j^k \leq \frac{1}{2}\left[b_i^l + b_i^u - \frac{4}{5}\alpha_i + \frac{2}{3}\beta_i\right], (i = 1,2,\dots,m) \\ \sum_{k=1}^{k_j} \beta_j^k = 1, & (j = 1,2,\dots,n) \\ \quad \beta_j^k \geq 0 \; for\ all\ j\ and\ k \end{cases} \tag{11.c}$$

such that

$$\sum_{i=1}^{k} w_i \left(f_i(x_i^*) + \sum_{j=1}^{n}(x_j - x_{ij}^*)\frac{\partial f_i(x_i^*)}{\partial x_j}\right) = \sum_{i=1}^{k} w_i \left(\sum_{j=1}^{n} c_j^i x_j\right) = \sum_{j=1}^{n} \hat{C}_j \left(\sum_{k=1}^{k_j} \beta_j^k \hat{x}_j^k\right)$$

Note that β_j^k are the decision variables of the modified problem and the optimal solution of the original problem can be obtained from the relation:

$$x_j^* = \sum_{k=1}^{k_j} \beta_j^k \hat{x}_j^k, (j = 1,2,\dots,n) \tag{12}$$

Finally, using the branch-and-bound method [7] to find the integer solution.

To test a point x^* to be efficient to FLSMLFP problems, using the following efficiency test and theorem (Chankong and Haimes (1983), [6].

Efficiency Test

To test a point $\bar{x}$ belongs to the feasible domain (3.b)-(3.d) to be efficient to problems (3.a)-(3.b), select $\bar{w} \in R^k$, $\bar{w}_i > 0$ $i \in \{1,2,\dots,k\}$, and solve the following problem:

$$P(\Psi) = max\Psi = \sum_{i=1}^{k} \bar{w}_i s_i \tag{13}$$

subject to

$$x \in S_M \;\; (or \;\; x \in S_Y)$$

$$f_i(x)s_i = f_i(\bar{x})s_i \geq 0, i \in \{1,2,\dots,k\}$$

Where $f_i(x)$ is the ith objective function in problems (3.a)-(3.b).

Theorem 1. For given $\bar{w} \in R^k$, $\bar{w} > 0$ and $\bar{x}$ in the feasible domain (3.b)-(3.d), let Ψ^* be an optimum value of the problem (13) then:

i. $\bar{x}$ is an efficient solution to problems (3.a)-(3.b), if $\Psi^* = 0$.

ii. The optimal solution of problem (13) is an efficient solution to problems (3.a)-(3.b), if $0 < \Psi^* < \infty$.

4. Numerical Example

The following example demonstrates the computational procedure for an FLSMLFP problem:

$$\text{FLSMLFP}: maxF(x) = \{f_1(x), f_2(x), f_3(x), f_4(x), f_5(x), f_6(x)\},$$
$$\text{subject to}$$
$$x_1 + x_2 + x_3 + x_4 \leq (15, 25, 10, 10),$$
$$5x_1 + x_2 \leq (4, 8, 2, 2),$$
$$x_3 + x_4 \geq (2, 3, 1, 1),$$
$$x_3 + 5x_4 \geq (20, 30, 15, 15),$$
$$x_1, x_2, x_3, x_4 \geq 0 \; and \; integer.$$

where

$$f_1(x) = \frac{x_1+x_2+x_3+x_4}{4x_1+3x_2+x_3+x_4+3}, \; f_2(x) = \frac{x_1+2x_2-x_3+x_4}{4x_1+3x_1+x_3+x_4+3}, \; f_3(x) = x_1 - x_2 - x_3 + x_4$$
$$f_4(x) = \frac{2x_1-x_2+x_3-x_4}{4x_1+3x_2+x_3+x_4+3}, \; f_5(x) = \frac{x_1+2x_2+x_3-x_4}{4x_1+3x_1+x_3+x_4+3}, \; f_6(x) = \frac{x_1+x_2-x_3-x_4}{4x_1+3x_1+x_3+x_4+3}$$

I. Using Maleki ranking function [11]

Using Maleki ranking function [11], the equivalent crisp model of FLSMLFP is written, as follows:

$$\text{LSMLFP}: maxF(x) = \{f_1(x), f_2(x), f_3(x), f_4(x), f_5(x), f_6(x)\},$$
$$\text{subject to}$$
$$x_1 + x_2 + x_3 + x_4 \leq 40,$$
$$5x_1 + x_2 \leq 12,$$
$$x_3 + x_4 \geq 5,$$
$$x_3 + 5x_4 \geq 50,$$
$$x_1, x_2, x_3, x_4 \geq 0 \; and \; integer.$$

Solving LSMLFP problem for each objective function one by one, then the optimal solution is $(x_1^*, x_2^*, x_3^*, x_4^*)_{f_1} = (0, 0, 38, 2)$, $f_1^* = 0.9302$, $(x_1^*, x_2^*, x_3^*, x_4^*)_{f_2} = (0, 0, 0, 10)$, $f_2^* = 0.7692$, $(x_1^*, x_2^*, x_3^*, x_4^*)_{f_3} = (2, 0, 0, 10)$, $f_3^* = 12$, $(x_1^*, x_2^*, x_3^*, x_4^*)_{f_4} = (0, 0, 40, 0)$, $f_4^* = 0.9302$, $(x_1^*, x_2^*, x_3^*, x_4^*)_{f_5} = (0, 0, 40, 0)$, $f_5^* = 0.9302$, $(x_1^*, x_2^*, x_3^*, x_4^*)_{f_6} = (0, 12, 3, 2)$ and $f_6^* = 0.1591$. Therefore, the objective functions are transformed by using the 1st order Taylor series [18] to linear functions as follows:

$f_1(x) \cong \overline{f}_1(x) = -0.0633x_1 - 0.0416x_2 + 0.0016x_3 + 0.0016x_4 + 0.8653,$
$f_2(x) \cong \overline{f}_2(x) = -0.1598x_1 - 0.0237x_2 - 0.1361x_3 + 0.0178x_4 + 0.5917,$
$f_3(x) = x_1 - x_2 - x_3 + x_4,$
$f_4(x) \cong \overline{f}_4(x) = -0.04x_1 - 0.0882x_2 + 0.0016x_3 - 0.0449x_4 + 0.8653,$
$f_5(x) \cong \overline{f}_5(x) = -0.0633x_1 - 0.0184x_2 + 0.0016x_3 - 0.0416x_4 + 0.8653,$
$f_6(x) \cong \overline{f}_6(x) = 0.0083x_1 + 0.0119x_2 - 0.0263x_3 - 0.0263x_4 + 0.1381.$

Using the nonnegative weighted sum approach [9]; let

$$w_1^* = 0.3, w_2^* = 0.2, w_3^* = 0.2, w_4^* = 0.1, w_5^* = 0.1 \text{ and } w_6^* = 0.1.$$

Then, the LSMLFP problem becomes large-scale single-objective integer linear programming and takes the form:

$$\text{LSLP: } maxP(x) = 0.1396x_1 - 0.2267x_2 - 0.2291x_3 + 0.1928x_4 + 0.5648,$$

subject to

$$x_1 + x_2 + x_3 + x_4 \leq 40,$$
$$5x_1 + x_2 \leq 12,$$
$$x_3 + x_4 \geq 5,$$
$$x_3 + 5x_4 \geq 50,$$
$$x_1, x_2, x_3, x_4 \geq 0 \; and \; integer.$$

Using the decomposition technique [5, 19] together with the branch-and-bound method [7], the optimal integer solution for the large-scale model given in the illustrated example is as follows: $X^* = (x_1^*, x_2^*, x_3^*, x_4^*) = (2, 0, 0, 10)$, $f_1^* = 0.5714$, $f_2^* = 0.5714$, $f_3^* = -8$, $f_4^* = -0.2857$, $f_5^* = -0.381$, $f_6^* = -0.381$ and $F^* = 2.772$.

Using the efficiency test, the optimal integer solution $X^* = (x_1^*, x_2^*, x_3^*, x_4^*) = (2, 0, 0, 10)$ is efficient.

II. Using Yager ranking function [21]

Using Yager ranking function [21], then the equivalent crisp model of FLSMLFP problem is written as:

$$\text{LSMLFP: } maxF(x) = \{f_1(x), f_2(x), f_3(x), f_4(x), f_5(x), f_6(x)\},$$

subject to

$$x_1 + x_2 + x_3 + x_4 \leq \frac{58}{3},$$
$$5x_1 + x_2 \leq \frac{88}{15},$$
$$x_3 + x_4 \geq \frac{73}{30},$$
$$x_3 + 5x_4 \geq 24,$$
$$x_1, x_2, x_3, x_4 \geq 0 \; and \; integer.$$

Applying the same previous steps, we get the efficient optimal integer solution is $X^* = (x_1^*, x_2^*, x_3^*, x_4^*) = (1, 0, 0, 4)$, $f_1^* = 0.4545$, $f_2^* = 0.4545$, $f_3^* = 5$, $f_4^* = -0..1818$, $f_5^* = -0.2727$, $f_6^* = -0.2727$ and $F^* = 1.4756$.

5. CONCLUSION

This study presents a method to tackle fuzzy large-scale multiple objective integer linear fractional programming (FLSMLFP) problem, using various kinds of linear ranking functions. In the crisp model, the first order Taylor series together with the weighting method can be used to formulate the LSLP problems. Then, the decomposition technique and the branch-and-bound method can be used to complete the solution process. To test the validity of this method, a numerical example is provided. The results obtained show the applicability and accuracy of proposed technique.

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